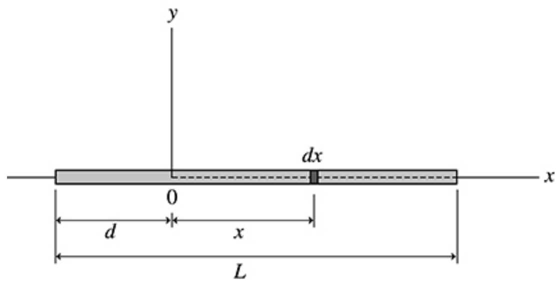
12.57. Visualize:



We chose the origin of the coordinate system to be on the axis of rotation, that is, at a distance d from one end of the rod.

Solve: The moment of inertia can be calculated as follows:

$$I = \int_{x_1}^{x_2} x^2\, dm \quad \text{and} \quad \frac{dm}{M} = \frac{dx}{L} \Rightarrow dm = \frac{M}{L} dx$$

$$\Rightarrow I = \frac{M}{L} \int_{-d}^{L-d} x^2 dx = \left(\frac{M}{L}\right) \frac{x^3}{3}\Bigg|_{-d}^{L-d} = \frac{1}{3}\left(\frac{M}{L}\right)[(L-d)^3 - (-d)^3] = \frac{M}{3L}[(L-d)^3 + d^3]$$

For $d = 0$ m, $I = \frac{1}{3}ML^2$, and for $d = \frac{1}{2}L$,

$$I = \frac{M}{3L}\left[\left(\frac{L}{2}\right)^3 + \left(\frac{L}{2}\right)^3\right] = \frac{1}{12}ML^2$$

Assess: The special cases $d = 0$ m and $d = L/2$ of the general formula give the same results that are found in Table 12.2.